

AFOSR 819

IMM-NYU 288
JUNE 1961



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INSTITUTE OF
MATHEMATICAL SCIENCES

A Machine Program For Theorem-Proving

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MATHEMATICAL SCIENCES DIRECTORATE
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IMM-288
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New York University
Institute of Mathematical Sciences
Mathematical Sciences Directorate
Air Force Office of Scientific Research
Washington 25, D.C.
AFOSR 819

A MACHINE PROGRAM FOR THEOREM-PROVING
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ABSTRACT: The programming of a proof procedure is discussed in connection with trial runs and possible improvements.

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The research reported in this document has been sponsored by the Mathematical Sciences Directorate, Air Force Office of Scientific Research, under Contract No. AF 49(638)-777.

A Machine Program for Theorem-Proving
Martin Davis, George Logemann, and Donald Loveland.

In [1] is set forth an algorithm for proving theorems of quantification theory which is an improvement in certain respects over previously available algorithms such as that of [2]. The present paper deals with the programming of the algorithm of [1] for the New York University, Institute of Mathematical Sciences' IBM 704 computer, with some modifications in the algorithm suggested by this work, with the results obtained using the completed algorithm. Familiarity with [1] is assumed throughout.

1. Changes in the algorithm and programming techniques used.

The algorithm of [1] consists of two interlocking parts. The first part, called the QFL-Generator, generates (from the formula whose proof is being attempted) a growing propositional calculus formula in conjunctive normal form, the "quantifier-free lines." The second part, the Processor, tests, at regular stages in its "growth," the consistency of this propositional calculus formula. An inconsistent set of quantifier-free lines constitutes a proof of the original formula.

The algorithm of [1] used in testing for consistency proceeded by successive elimination of atomic formulas, first eliminating one-literal clauses (one-literal clause rule), and then atomic formulas all of whose occurrences were positive or all of whose occurrences were negative (affirmative-negative rule). Finally, the remaining atomic formulas were to have been eliminated by the rule:

III. Rule for Eliminating Atomic Formulas. Let the given formula F be put into the form $(A \vee p) \& (B \vee \bar{p}) \& R$ where A , B , and R are free of p . (This can be done simply by grouping

together the clauses containing p and "factoring out" occurrences of p to obtain A , grouping the clauses containing \bar{p} and "factoring out" \bar{p} to obtain B , and grouping the remaining clauses to obtain R .) Then F is inconsistent if and only if $(A \vee B) \wedge R$ is inconsistent.

After programming the algorithm using this form of Rule III, it was decided to replace it by the following rule:

III*. Splitting Rule. Let the given formula F be put in the form $(A \vee p) \wedge (B \vee \bar{p}) \wedge R$ where A, B , and R are free of p . Then F is inconsistent if and only if $A \wedge R$ and $B \wedge R$ are both inconsistent.

Justification of Rule III*. For¹ $p = 0$, $F = A \wedge R$; for $p = 1$, $F = B \wedge R$.

The forms of Rule III are interchangeable; although theoretically they are equivalent, in actual applications each has certain desirable features. We used Rule III* because of the fact that Rule III can easily increase the number and the lengths of the clauses in the expression rather quickly after several applications. This is prohibitive in a computer if one's fast access storage is limited. Also, it was observed that after performing Rule III, many duplicated and thus redundant clauses were present. Some success was had by causing the machine to systematically eliminate the redundancy; but the problem of total length increasing rapidly still remained when more complicated problems were attempted. Also use of Rule III can seldom yield new one-literal clauses, whereas use of Rule III* often will.

In programming Rule III*, we used auxiliary tape storage. The rest of the testing for consistency is carried out using only fast access storage. When the "Splitting Rule" is used one of the two formulas resulting is placed on tape. Tape memory records are organized in the cafeteria stack-of-plates scheme: the last record written is the first to be read.

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In the program written for the IBM 704, the matrix and conjunction of quantifier-free lines are coded into cross-referenced associated (or linked) memory tables by the QFL-Generator and then analyzed by the Processor. In particular, the QFL-Generator is programmed to read in the matrix M in suitably coded Polish (i.e., "parenthesis-free") form. The conversion to a quantifier-free matrix in conjunctive normal form requires, of course, a certain amount of pencil work on the formula, which could have been done by the computer. In doing this, we departed from [1], by not using prenex normal form. The steps are :

(1) Write all truth-functional connectives in terms of \sim , $\&$, \vee .

(2) Move all \sim 's inward successively (using de Morgan laws) until they either are cancelled (with another \sim) or acting on an atomic formula.

(3) Now, replace existential quantifiers by function symbols (cf. [1], p. 205), drop universal quantifiers, and place in conjunctive normal form. A simple one-to-one assembler was written to perform the final translation of the matrix M into octal numbers.

It will be recalled that the generation of quantifier-free lines is accomplished by successive substitutions of "constants" for the variables in the matrix M. In the program the constants are represented by the successive positive integers.

For a matrix containing n individual variables, the n-tuples of positive integers are generated in a sequence of increasing norm such that all n-tuples with a given norm are in decreasing n-ary numerical order. Here we define the norm of $(j_1, \dots, j_n) = j_1 + \dots + j_n = ||j_i||$. Other norms could have been used.

For example, Gilmore [2] takes for $||j_i||$, the maximum of j_1, \dots, j_n . In [1] more complicated norm is indicated.

Substitutions of successive n-tuples into the matrix cause

new constants to appear in the matrix. The program numbers constants in their order of appearance. Thus, the constants are ordered by the program in a manner depending upon the input data. By rearranging the clauses of a formula a different order would in general be created. In some cases, whether or not the program could actually prove the validity of a given formula (without running out of fast access storage) depended on how one shuffled the punched-data deck before reading it into the assembler ! Thus, the variation in ordering of constants did affect by a factor of 10 (from 500 to 5000) the number of lines needed to prove the validity of :

$$(e)(Ed)(x)(y)[S(x,y,d) \longrightarrow T(x,y,e)]$$

$$\longrightarrow (e)(x)(Ed)(y)[S(x,y,d) \longrightarrow T(x,y,e)]$$

(This valid formula may be thought of as asserting that uniform continuity implies continuity if we set:

$$S(x,y,d) \longleftrightarrow |x - y| < d$$

$$T(x,y,e) \longleftrightarrow |f(x) - f(y)| < e.)$$

In storing the quantifier-free lines, two tables are used. The first, called the conjunction table, is a literal image of the quantifier-free lines in which one machine word corresponds to one literal, i.e. to p or $\sim p$ where p is an atomic formula. The lines in the second, or formula table are themselves heads of two chain lists giving the occurrences of p and $\sim p$ respectively in the conjunction table. In addition, included for formula p in the formula table are counts of the number of clauses in which p and $\sim p$ occur and total number of all literals in these clauses; the formula table is itself a two-way linked list. A third short list of those literals is kept in which are entered all formulas to which the one-literal clause and affirmative-negative rules apply; this is called the ready list.

If the program tries to enter p and $\sim p$ into the ready list, an inconsistency has been found; the machine stops.

The totality of the processing rules requires only two basic operations: a subroutine to delete the occurrences of a literal p or $\sim p$ from the quantifier-free lines, and a routine to eliminate from them all the clauses in which p or $\sim p$ occur.

We may observe that only the deletion program can create new one-literal clauses, and likewise applications of the affirmative-negative rule can come only from the elimination program.

The machine thus performs the one literal-clause and affirmative-negative rules as directed by the ready list until the ready list is empty. It is possible that the choice of p to be eliminated first is quite critical in determining the length of computation required to reach a conclusion: a program to choose p is used, but no tests were made to vary this segment of the program beyond a random selection, namely the first entry in the formula table. To perform Rule III*, one saves the appropriate tables with some added reference information in a tape record, then performs an elimination on $\sim p$ and a deletion on p . At a consequent discovery of consistency, one must generate more quantifier free lines; the QFL-generator is recalled. Otherwise, at finding an inconsistency, the machine must check to see if there are any records on the Rule III* tape: if none, the quantifier-free lines were inconsistent; otherwise, it reads in the last record.

If one uses Rule III, (which we did in an early version of our program) an entirely different code is needed. The problem is precisely that of mechanizing the application of the distributive law.

2. Results obtained in running the program.

At the time the programming of the algorithm was undertaken, we hoped that some mathematically meaningful and, perhaps non-trivial, theorems could be proved. The actual achievements in this direction were somewhat disappointing. However, the program

performed as well as expected on the simple predicate calculus formulas offered as fare for a previous proof procedure program.

(See Gilmore [1].) In particular, the well-formed formula

$$(Ex)(Ey)(z) \left\{ F(x,y) \rightarrow (F(y,z) \& F(z,z)) \& ((F(x,y) \& G(x,y)) \rightarrow (G(x,z) \& G(z,z))) \right\}$$

which was beyond the scope of Gilmore's program was proved in under two minutes with the present program. Gilmore's program was halted at the end of 7 "substitutions", (quantifier-free lines) after an elapsed period of about 21 minutes. It was necessary for the present program to generate approximately 60 quantifier-free lines before the inconsistency appeared.² Indeed, the "uniform continuity implies continuity" example mentioned above required over 500 quantifier-free lines to be generated and was shown to be valid in just over two minutes. This was accomplished by nearly filling the machine to capacity with generated quantifier-free lines (2000 lines in this case) before applying any of the rules of reduction .

Rather than describe further successes of the program, it will be instructive to consider in detail a theorem that the program was incapable of proving and to examine the cause for this. This particular example is one the authors originally had hoped the program could prove, an elementary group theory problem. In essence, it is to show that in a group a left inverse is also a right inverse.

It is, in fact, quite easy to follow the behavior of the

proof procedure on this particular example as it parallels the usual approach to the problem. The problem may be formulated as follows :

- Axioms:
1. $e \cdot x = x$
 2. $I(x) \cdot x = e$
 3. $(x \cdot y) \cdot z = w \Rightarrow x \cdot (y \cdot z) = w$
 4. $x \cdot (y \cdot z) = w \Rightarrow (x \cdot y) \cdot z = w$

Conclusion: $x \cdot I(x) = e$

The letter e is interpreted as the identity element and the function I as the inverse function. The associative law has been split into two clauses for convenience.

A proof is as follows :

1. $I(I(x)) \cdot I(x) = e$ by Axiom 2
2. $e \cdot x = x$ by Axiom 1
3. $I(x) \cdot x = e$ by Axiom 2
4. $I(I(x)) \cdot e = x$ by Axiom 3, taking $(I(I(x)), I(x), x)$ for (x, y, z)
5. $e \cdot I(x) = I(x)$ by Axiom 1
6. $I(I(x)) \cdot I(x) = e$ by Axiom 2
7. $I(I(x)) \cdot e = x$ step 4
8. $x \cdot I(x) = e$ by Axiom 4, taking $(I(I(x)), e, I(x))$ for (x, y, z)

Step 8. is the desired result.

To formalize this proof would require adjoining axioms of equality. To avoid this, one can introduce the predicate of

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial matters.

2. The second part outlines the specific procedures for recording transactions. It states that all transactions must be recorded in a clear and concise manner, using standardized formats and codes. This ensures that the information is easily accessible and understandable to all relevant parties.

3. The third part addresses the issue of data security and confidentiality. It stresses that all recorded information must be protected from unauthorized access and disclosure. Appropriate security measures, such as encryption and access controls, should be implemented to safeguard the data.

4. The fourth part discusses the role of technology in record-keeping. It notes that modern information systems can significantly enhance the efficiency and accuracy of record-keeping processes. However, it also warns against over-reliance on technology and emphasizes the need for human oversight and verification.

5. The fifth part covers the topic of data retention and archiving. It specifies that records should be retained for a minimum of five years, unless otherwise specified by applicable laws or regulations. Proper archiving procedures should be followed to ensure the long-term preservation and retrievability of the data.

6. The sixth part discusses the importance of regular audits and reviews. It states that periodic audits are necessary to verify the accuracy and completeness of the records. These audits should be conducted by independent parties to ensure objectivity and fairness.

7. The seventh part addresses the issue of data sharing and collaboration. It encourages the use of secure channels for sharing information between different departments or organizations, while maintaining strict controls to prevent data leakage.

8. The eighth part discusses the role of training and education in ensuring effective record-keeping. It emphasizes that all personnel involved in the process must receive appropriate training and education to understand the importance of accurate record-keeping and the correct procedures to follow.

9. The ninth part covers the topic of data backup and recovery. It states that regular backups of all recorded data should be performed to prevent data loss in the event of a system failure or disaster. A robust recovery plan should also be in place to ensure that the data can be restored quickly and accurately.

10. The tenth and final part discusses the overall goals and objectives of the record-keeping system. It states that the primary goal is to ensure the integrity, accuracy, and availability of all recorded information, thereby supporting the organization's operations and decision-making processes.

three arguments $P(x,y,z)$, interpreted as $x \cdot y = z$. The theorem reformulated becomes

- Axioms:
1. $P(e,x,x)$
 2. $P(I(x),x,e)$
 3. $\sim P(x,y,u) \vee \sim P(u,z,w) \vee \sim P(y,z,v) \vee P(x,v,w)$
 4. $\sim P(y,z,v) \vee \sim P(x,v,w) \vee \sim P(x,y,u) \vee P(u,z,w)$

Conclusion : $P(x,I(x),e)$.

The theorem to be proved valid is the implication of the conjunction of the four axioms with the conclusion, the universal quantifiers appearing outside the matrix.

To complete the preparation of the well-formed formula for encoding for the assembler, it is necessary to negate the conclusion. (cf. [1], p. 204).

The single existential quantifier has no dependence on the universal quantifiers, hence leads to the constant function s when this existential quantifier is replaced by a function symbol. (cf. [1], p. 205.)

The conclusion then becomes

$$\sim P(s, I(s), e) .$$

The conjunction of this with the 4 axioms gives the desired form.

As seen from the proof previously noted the quantifier-free clauses needed to produce the inconsistency are

1. $P(I(I(s)), I(s), e)$
2. $P(e, s, s)$
3. $P(I(s), s, e)$

[illegible]

1. *Chlorophyll *a** and *Chlorophyll *b** were determined by the method of Arar and Collins (1971) using a Shimadzu 1601 spectrophotometer.

1. *Chlorophyll a* and *Chlorophyll b* were determined by the method of Arar and Collins (1971) using a Shimadzu 1010 spectrophotometer.

Journal of Management Studies, 19(6), 701-718.

[illegible]

1. *Chlorophyll a* and *Chlorophyll b* were determined by the method of Arar and Collins (1971) using a Shimadzu 1010 spectrophotometer.

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Journal of Management Studies, 19(6), 701-718.

4. $\sim P(I(I(s)), I(s), e) \vee \sim P(e, s, s) \vee \sim P(I(s), s, e) \vee P(I(I(s)), e, s)$
5. $P(e, I(s), I(s))$
6. $\sim P(e, I(s), I(s)) \vee \sim P(I(I(s)), I(s), e) \vee \sim P(I(I(s)), e, s) \vee P(s, I(s), e)$
7. $\sim P(s, I(s), e)$

(It is quite clear in this case that successive applications of the one-literal clause rule reduce this set to:

$$P(s, I(s), e) \& \sim P(s, I(s), e).)$$

The question to be considered is: how many quantifier-free lines must be generated by the present program to realize these required lines? The constants are generated in the following order:

1. e
2. s
3. $I(s)$
4. $I(e)$
5. $I(I(s))$
- etc.

(The constants are identified directly with their index e.g. the 6-tuple (1,1,1,1,1,1) represents (e,e,e,e,e,e). As this is the first substitution, the program assigns in order, reading the well-formed formula backwards and from the inside out for nested functions; $e, s, I(s), I(e), I(I(s))$. The $I(I(s))$ appears when x is assigned $I(s)$, no new entries occurring until this time. Note that there are 6 free variables (u, v, w, x, y, z)

1. The first step is to identify the problem and its scope.

2. The second step is to gather data.

3. The third step is to analyze the data.

4.

5. The fifth step is to implement the solution.

6. The sixth step is to evaluate the results.

7. The seventh step is to document the process.

8. The eighth step is to review the process.

9. The ninth step is to improve the process.

10. The tenth step is to maintain the process.

11. The eleventh step is to monitor the process.

12. The twelfth step is to report the results.

13. The thirteenth step is to conclude the process.

14. The fourteenth step is to end the process.

15. The fifteenth step is to finish the process.

16. The sixteenth step is to complete the process.

17. The seventeenth step is to close the process.

18. The eighteenth step is to terminate the process.

19. The nineteenth step is to stop the process.

20. The twentieth step is to end the process.

21. The twenty-first step is to finish the process.

22. The twenty-second step is to complete the process.

23. The twenty-third step is to close the process.

24. The twenty-fourth step is to terminate the process.

25. The twenty-fifth step is to stop the process.

in the matrix).

The program generates the needed n-tuples by producing all possible n-tuples of integers whose sum N of entries is fixed, $N=n, n+1, \dots$. Thus it is only necessary to consider the n-tuple which has the maximum sum of entries. In this case, the substitution $u = s, v = I(s), w = e, x = I(I(s)), y = e, z = I(s)$ (required for axiom 4 to produce the clause 6 in the "proof" above in a quantifier-free line) gives the n-tuple with maximum sum. The n-tuple is seen to be (2,3,1,5,1,3), the sum equals 15. The combinatorial expression $\binom{N}{n}$ gives the total number of n-tuples of positive integers whose sum is less than or equal to N.³

It is seen that to prove this theorem at least $\binom{14}{6} = 3003$ lines must be generated and that the inconsistency will be found on or before $\binom{15}{6} = 5005$ lines have been generated.⁴

The present program generated approximately 1300 quantifier free lines. This number of quantifier-free lines was accomplished holding all major tables simultaneously in core memory, limited to 32,768 "words". (This was done to insure a reasonable time factor for any problem within possible scope of the program.

For this reason also, the entire program was coded in SAP with many time-saving devices employed.)

The authors feel that a reprogramming to make use of tape storage of tables might realize a factor of 4 for the total number of quantifier-free lines attainable before running time became prohibitive. This would be just sufficient for this problem. That realizing this extra capacity is really uninter-

The first part of the paper is devoted to the

study of the properties of the function

defined on the interval $[0, 1]$ by the formula

where α is a real number, $0 < \alpha < 1$.

It is known that this function is continuous on

the interval $[0, 1]$ and that it is differentiable

at the point $x = 0$ if and only if $\alpha > 1$.

In the present paper we shall study the

properties of the function $f(x)$ for

arbitrary values of α and shall prove

that the function $f(x)$ is differentiable

at the point $x = 0$ if and only if

$$\alpha > \frac{1}{2}.$$

The results obtained in this paper are

contained in the following theorem.

Theorem 1. Let $f(x)$ be the function

defined on the interval $[0, 1]$ by the formula

where α is a real number, $0 < \alpha < 1$.

Then the function $f(x)$ is differentiable

at the point $x = 0$ if and only if

$\alpha > \frac{1}{2}$.

The proof of this theorem is given in

the following sections of the paper.

In the first section we shall prove

that the function $f(x)$ is

esting is seen by noting that if the conclusion was placed before the axioms, altering the validity of the matrix not at all, the element $I(e)$ would be generated before $I(s)$ and the needed n -tuple would sum to 16. Then $\binom{16}{6} = 8008$ becomes the upper bound, beyond the capacity of the projected program. Other formulations of the same problem result in quite unapproachable figures for the number of quantifier-free lines needed. (For another example illustrating the same situation, see Prawitz [3]).

The existing program allows one to think of working with a capacity of 1000 or 2000 quantifier-free lines instead of a capacity of 10 or 20, the previous limit. The time required to generate additional quantifier-free lines is independent of the number of quantifier-free lines already existing. Against this linear growth of number of quantifier-free lines generated, there is, in a meaningful sense, an extreme non-linear growth in the number of quantifier-free lines to be considered with increasingly more "difficult" problems. This is true of simple enumeration schemes of the nature considered here. It seems that the most fruitful future results will come from reducing the number of quantifier-free lines that need be considered, by excluding, in some sense, "irrelevant" quantifier-free lines. Some investigation in this area has already been done (see Prawitz [3]).

Footnotes.

¹ As in [1], 1 stands for "truth", and 0 for "falsehood".

² In [1], a hand-calculation of this example using the present scheme showed inconsistency at 25 quantifier-free lines. The discrepancy is due to a different rule for generation of constants.

³ To see this, consider a sequence of $N + 1$ ones. Flag n of these. The flag is to be interpreted "sum all 1's, including the flagged 1, to the next flag and consider this sum as an entry in the n -tuple". Placing an unflagged 1 on the extreme left, leaving it fixed, consider the possible permutations of all other symbols. The different sequences total $\binom{N}{n}$ and, regarding the set of 1's starting with the last flagged 1 as overflow, this is seen to represent precisely the desired n -tuples.

⁴ If the rule for generating n -tuples had been, for each m , to generate all n -tuples possible such that each entry assumes a positive integral value $\leq m$, it is clear that at least $4^6 = 4096$ quantifier-free lines would be needed and $5^6 = 15625$ lines would suffice to guarantee a solution. If no more information were available, one sees an intuitive advantage, in this case, for using the previous method. In general, the authors see no preference for either method, in contrast to some previous suggestions that the latter method seemed intuitively better.

Introduction

The purpose of this report is to provide a detailed analysis of the data collected during the experiment.

The data was collected from a series of experiments conducted over a period of six months.

The results of the experiments are presented in the following sections.

The first section describes the experimental setup and the methods used to collect the data.

The second section presents the results of the experiments, including the mean values and standard deviations.

The third section discusses the statistical analysis of the data, including the use of t-tests and ANOVA.

The fourth section provides a summary of the findings and conclusions drawn from the data.

The fifth section discusses the limitations of the study and suggests areas for future research.

The sixth section provides a conclusion and a summary of the key findings.

The seventh section provides a list of references and a bibliography.

The eighth section provides a list of figures and tables.

The ninth section provides a list of appendices.

The tenth section provides a list of acknowledgments.

The eleventh section provides a list of contact information.

The twelfth section provides a list of funding sources.

The thirteenth section provides a list of other relevant information.

The fourteenth section provides a list of other relevant information.

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The sixteenth section provides a list of other relevant information.

The seventeenth section provides a list of other relevant information.

The eighteenth section provides a list of other relevant information.

The nineteenth section provides a list of other relevant information.

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THE

1. The first part of the book is devoted to a general discussion of the theory of the atom, and to a description of the various experiments which have been performed in order to determine the structure of the atom. (1)
2. The second part of the book is devoted to a description of the various experiments which have been performed in order to determine the structure of the atom. (2)
3. The third part of the book is devoted to a description of the various experiments which have been performed in order to determine the structure of the atom. (3)
4. The fourth part of the book is devoted to a description of the various experiments which have been performed in order to determine the structure of the atom. (4)
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9. The ninth part of the book is devoted to a description of the various experiments which have been performed in order to determine the structure of the atom. (9)
10. The tenth part of the book is devoted to a description of the various experiments which have been performed in order to determine the structure of the atom. (10)

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1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the transparency and accountability of the organization. This section also outlines the various methods used to collect and analyze data, ensuring that the information is reliable and up-to-date.

2. The second part of the document focuses on the implementation of the proposed changes. It details the steps involved in the transition process, from the initial planning phase to the final execution. This section also addresses the potential challenges that may arise during the implementation and provides strategies to overcome them.

3. The third part of the document discusses the impact of the proposed changes on the organization's overall performance. It highlights the expected benefits, such as increased efficiency and cost savings, and provides a detailed analysis of the potential risks. This section also includes a comparison of the current state of the organization with the proposed changes, illustrating the expected improvements.

4. The fourth part of the document provides a summary of the key findings and conclusions. It reiterates the importance of the proposed changes and the need for continued monitoring and evaluation. This section also includes a list of recommendations for future actions, ensuring that the organization remains committed to the principles of transparency and accountability.

5. The fifth part of the document is a conclusion, summarizing the main points of the document and expressing the author's confidence in the proposed changes. It also includes a statement of the author's commitment to the organization's success and a final note of appreciation for the support and cooperation of all stakeholders.

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